Sequence and series

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1) **Sequence & Series**

$\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \ldots$ ← Sequence.

Expression for **general term**.

$$u_n = \frac{n}{2n+1}$$

$n = 3, \quad u_3 = \frac{3}{2(3)+1} = \frac{3}{7}$

$n = 4, \quad u_4 = \frac{4}{8+1} = \frac{4}{9}$

1) **Arithmetic sequence**. ← difference is constant
2) Geometry sequence. ← Ratio is constant.

1) 2, 4, 6, 8, 10, 12 - - - - -

2 2 2 2 2

Arithmetic Sequence.

Common difference = $4-2 \neq 2 = 10-8 = 12-10$

2) $2^0, 2^1, 2^2, 2^3, - - - -$

$r_1 = \frac{2^1}{2^0} = 2, \quad r_3 = \frac{2^3}{2^2} = 2$

$r_2 = \frac{2^2}{2^1} = 2$ /

Ratio = 2 Geometric sequence.

Ex $u_n = 2u_{n-1} - 3$ and $u_1 = 1$

1, −1, −5, −13, −29, ---

Series:

$2 + 4 + 6 + 8 + 10$ --- ← Series:

Summation (sigma) Σ

Ex $\sum_{n=2}^{6} (4n-3) = 5 + 9 + 13 + 17 + 21$

$=$

$n = 2$ to $n = 6$

2, 3, 4, 5, 6

Ex $-3 + 6 - 12 + 24$

General term $u_n = 3(-1)^{n+1} \cdot 2^n$

−3, 6, −12, 24

n = 0, n = 1, n = 2, n = 3

3, 6, 12, 24

3×1, 3×2, 3×4, 3×8

2^0, 2^1, 2^2, 2^3

$u_n = 3(-1)^{n+1} \cdot 2^n$

$\sum_{n=0}^{3} 3(-1)^{n+1} \cdot 2^n = -3 + 6 - 12 + \underbrace{3(-1)^{3+1} \cdot 2^3}_{24}$

sigma.

Ex $8 + 12 + 16 + 20 + \cdots$

$\sum_{n=1}^{\infty} 4n+4$, $\sum_{n=2}^{\infty} 4n$

Arithmetic sequence (A.P) progression.

1) $2, 7, 12, \ldots\ldots$ u_9

Common difference = 5 (d)

Common diff. = d.

$= u_1$

$u_2 = u_1 + d$ ⇐ 2nd

$u_3 = u_1 + 2d$ ⇐ 3rd

$u_4 = u_1 + 3d$ ⇐ 4th

$\vdots$

$u_n = u_1 + (n-1)d$ ⇐ n^{th}.

First term

common difference.

$u_9 = 2 + (9-1)\,5$

$= 2 + 8 \times 5$

$= 2 + 40 = 42$

Ex $3a, 6a, 9a, \ldots\ldots u_{12}$

Common diff. $= 6a - 3a = 9a - 6a$

$= 3a$

$u_{12} = 3a + (12-1)\,3a$

$= 3a + 11 \times 3a = 36a.$

Ex Given that two terms of an Arithmetic sequence are $u_5 = -3.7$ & $u_{15} = -52.3$

find the value of the 19th term.

find the value of the 19th term

Sol:

$$u_n = u_1 + (n-1)d$$

u_1 = first term
d = common diff

$$u_5 = u_1 + (5-1)d$$

$$-3.7 = u_1 + 4d \quad \text{— (1)}$$

$$u_{15} = u_1 + 14d$$

$$-52.3 = u_1 + 14d \quad \text{— (2)}$$

$$u_1 = 15.74 \ , \ d = -4.86$$

$$u_{19} = 15.74 + 18 \times (-4.86) =$$

$$u_{19} = u_1 + (19-1)d$$

Ex A movie theatre with 12 rows of seating has 30 seats in the first row. Each row behind it has two additional seats. How many seats are in the last row?

Hw

Geometric Sequences:-

common ratio $= r$

$$u_1$$
$$u_2 = u_1 r \qquad \frac{u_2}{u_1} = \frac{u_1 r}{u_1} = r$$
$$u_3 = u_1 r^2 \qquad \frac{u_3}{u_2} = \frac{u_1 r^2}{u_1 r} = r$$
$$u_4 = u_1 r^3$$
$$\vdots$$

$$u_n = u_1 r^{(n-1)}$$ ← first term, common ratio.

Ex $1, \ -\frac{1}{}, \ \frac{1}{}, \ \ldots, \ u_9$

Ex $\frac{1}{2}, \frac{-1}{4}, \frac{1}{8}, \ldots, u_9$

$$\text{ratio} = \frac{-1/4}{1/2} = -1/2$$

$$\text{ratio} = \frac{1/8}{-1/4} = -1/2$$

$$r = -1/2$$

$$u_9 = \frac{1}{2} \times \left(-\frac{1}{2}\right)^{9-1} = \frac{1}{512}$$

Ex If $16, x+2, 1$ are the first three terms of a geometric sequence, find all possible values of x.

Soln

$$\frac{x+2}{16} = \frac{1}{x+2}$$

$$\Rightarrow (x+2)^2 = 16 \times 1$$

$$\Rightarrow x+2 = \pm 4$$

$$x = -2 \pm 4.$$

$$x = -2+4, \ -2-4$$

$$x = 2, \ -6$$

$$r = \frac{\pm 4}{16} = \pm \frac{1}{4}$$

$(x+2)^2 = 16 \times 1$