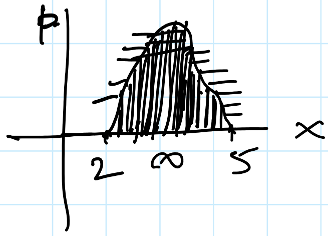
Mean and variance of continuous random variable

Tuesday, January 12, 2021 5:02 PM

$\mu = E(x) = \sum p.x.$ ← Discrete.

$\mu = E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$ ← continuous.



Ex

$f(x) = \frac{1}{2}(x-3)$ for $3 \le x \le 5$. Find E(X).

$$E(x) = \int_3^5 x\, f(x)\, dx = \int_3^5 x \cdot \frac{1}{2}(x-3)\, dx.$$

$$= \frac{1}{2}\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_3^5$$

$$E(x) = \frac{1}{2}\left\{\frac{125}{3} - \frac{75}{2} - \left(\frac{27}{3} - \frac{27}{2}\right)\right\}$$

$$= \frac{1}{2}\left\{\frac{98}{3} - \frac{48}{2}\right\} =$$

$$= \frac{1}{2}\left\{\frac{\quad}{6}\right\} = 4.33$$

$Var(x) = E(x^2) - [E(x)]^2$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx.$$

Find the standard deviation for the pdf in Example 1.

$$E(x^2) = \frac{1}{2}\int^5 x^2 \cdot (x-3)\, dx$$

3

$$= \frac{1}{2}\left[\frac{x^4}{4} - x^3\right]_3^5$$

$$= 19$$

$$var(X) = 19 - \left(\frac{13}{3}\right)^2 = \frac{2}{9}$$

$$\sigma = \sqrt{\frac{2}{9}} = \underline{\underline{0.471}}$$

Ex

The continuous random variable X has probability density function

$f(x) = kx$ for $0 \le x \le 5$

a) Find the value of k.

b) Find the mean and variance of X.

c) Calculate

i) $P(X > \mu)$ and

ii) $P(X > \mu + \sigma)$ where μ is the mean and σ is the standard deviation of X.

$$f(x) = \frac{2x}{25}$$

① $f(x) \ge 0$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

Soln

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

$$\int_0^5 kx\,dx = 1.$$

$$\Rightarrow \left[k\frac{x^2}{2}\right]_0^5 = 1$$

$$\Rightarrow \frac{25}{2}k = 1$$

$$\Rightarrow \boxed{k = \frac{2}{25}}$$

$$E(X) = \int_0^5 x\,f(x)\,dx$$

$$= \int_0^5 \frac{2}{25}x^2\,dx.$$

$$= \left[\frac{2}{25}\frac{x^3}{3}\right]_0^5$$

$$= \frac{2}{\cancel{25}}\left[\frac{\cancel{125}^{5}}{3}\right]$$

$$= \frac{10}{3}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$var(X) = \underline{\underline{1.4}} = \frac{25}{18}$$

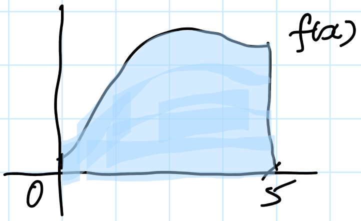
5

$$P(X = \mu) = 0$$

$P(X=\mu)=0$

c) $P(X>\mu) = \int_{10/3}^{5} f(x)\,dx$

$= \frac{5}{9}$ ✓



(ii) $\boxed{P(X \geq \mu+\sigma)} = \int_{4.5}^{5} f(x)\,dx$

$= \underline{\underline{0.186}}$

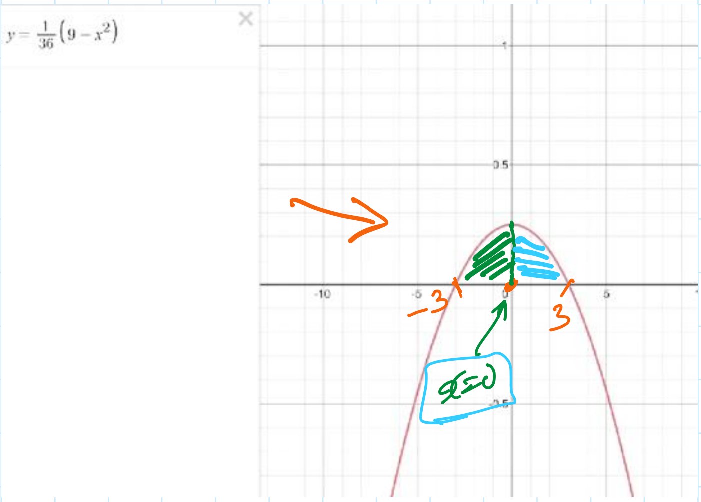
Ex

The continuous random variable X has probability density function

$f(x) = \frac{1}{36}(9-x^2)$ for $-3 \leq x \leq 3$

a) Find the mean and variance of X.

b) Calculate i) $P(X > 2)$ and ii) $P(|X| > \sigma)$, where σ is the standard deviation of X.



$E(X) = \int_{-3}^{3} x\,f(x)\,dx$

$= \int_{-3}^{3} \frac{x}{36}\cdot(9-x^2)\,dx$

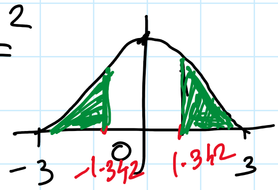
$= \frac{1}{36}\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_{-3}^{3}$

even

$f(x) = f(-x)$

$= \frac{1}{36}(0) = 0$

$Var = \frac{9}{5} = 1.8$, $\sigma = \sqrt{1.8} = \underline{\underline{1.342}}$



b) (i) $P(X>2) = \frac{2}{27}$

$|x| > \sigma$

(ii) $P(|X| > \sigma)$

$\boxed{|x| > 1.342}$

$P(|X| > 1.342)$

$\left.\begin{array}{l} 1.342 < X \\ x < -1.342 \end{array}\right\} |x| > 1.342$

$P(|X| > 1.342)$

$= P(X > 1.342 \ \& \ X < -1.342)$

$$P_1 = \int_{1.342}^{3} \frac{1}{36}(9 - x^2)\,dx.$$

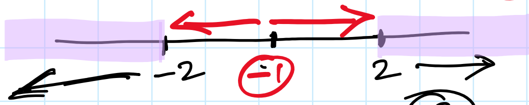
$= 0.18695$

$P = 2 \times P_1$

$= \underline{0.379}$

$\boxed{x < -1.342}$ } $|x| > 1.342$

$\boxed{|x| > 2}$



$-5, -4, -3$

$|-3| = 3 > 2$

3

$|3| > 2$

$3 > 2$

true.

$|-1| = 1 \not> 2$

$\left.\begin{array}{l} x > 2 \\ x < -2 \end{array}\right\}$