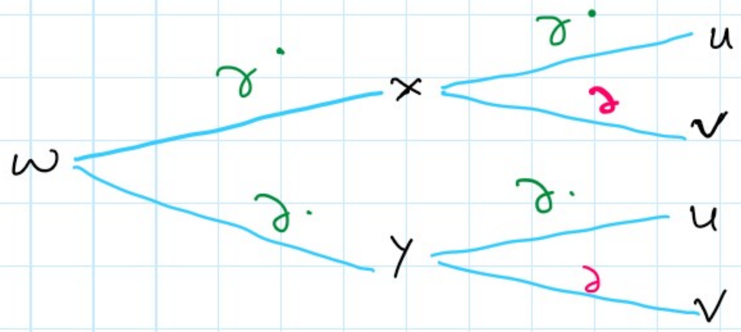
Chain rule

Wednesday, June 10, 2020 11:00 AM

B:- more than one independent variable:-



Main f^n. Intermediate variable. Independent variable.

Either find $\frac{\partial w}{\partial u}$ or $\frac{\partial w}{\partial v}$

✓ $$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \times \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial u}$$

✓ $$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial v}$$

Ex: $w = x^3 + y^3$, $x = u^2 + v^2$, $y = 2uv$

$$\frac{\partial w}{\partial u} = 6x^2u + 6y^2v$$

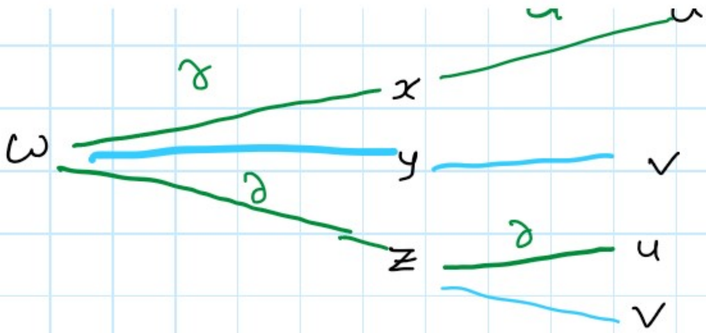
$$\frac{\partial w}{\partial v} = 6x^2v + 6y^2u$$

Ex: $w = x \tan^{-1} yz$, $x = u^{1/2}$, $y = e^{-2v}$, $z = v\cos u$

$$\frac{\partial w}{\partial u} = \frac{\tan^{-1}(yz)}{2\sqrt{u}} - \frac{xyv\sin u}{1+y^2z^2}$$

✓ $$\frac{\partial w}{\partial v} = \frac{-2xze^{-2v} + xy\cos u}{1+y^2z^2}$$ ✓

d u



$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial y}\cdot\frac{dy}{dv} + \frac{\partial w}{\partial z}\,\frac{\partial z}{\partial v}$$

Let

$$w = f(x,y) = 0$$

where $f(x,y)$ is a function such that $y = g(x)$ implicitly defined & x is the only independent variable.

$$\frac{\partial w}{\partial x} = \frac{\partial f}{\partial x}\cdot\underbrace{\frac{dx}{dx}}_{=1} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dx} = 0$$

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{\partial f}{\partial y}\cdot\frac{dy}{dx} = -\frac{\partial f}{\partial x}.$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\dfrac{\partial f}{\partial x}}{\dfrac{\partial f}{\partial y}} = -\frac{f_x}{f_y}$$

Ex: ✓ $$2x^2 + 3(xy)^{1/2} - 2y - 4 = 0$$

find $\frac{dy}{dx} = ?$

Sol^n

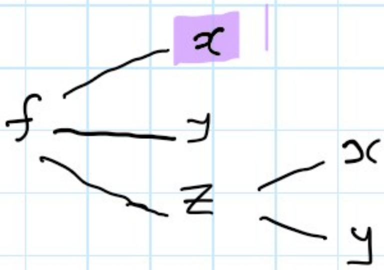
$$\frac{dy}{dx} = -\frac{8x\sqrt{xy} + 3y}{3x - 4\sqrt{xy}}$$

E: $w = f(x,y,z) = 0$ for $\boxed{z = g(x,y)}$

$$\frac{\partial z}{\partial x} = -\frac{f_x}{f_z} \quad , \quad \frac{\partial z}{\partial y} = -\frac{f_y}{f_z}$$

$$w = \underline{f(x, y, z)} = 0.$$

$$\frac{\partial w}{\partial x} = \frac{\partial f}{\partial x} \cdot \underbrace{\frac{\partial x}{\partial x}}_{=1} + \frac{\partial f}{\partial y} \cdot \underbrace{\frac{\partial y}{\partial x}}_{=0} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial x} = 0.$$



$$\Rightarrow \frac{\partial f}{\partial x} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial x} = 0$$

$$\Rightarrow \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial x} = -\frac{\partial f}{\partial x}.$$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}} = -\frac{f_x}{f_z}$$

$f(x, y, z)$

$$\frac{\partial z}{\partial y} = ?\ -\frac{f_y}{f_z}$$

$$w = f(x, y, z) = 0. \qquad z = g(x, y)$$

$$\frac{\partial w}{\partial y} = \frac{\partial f}{\partial x} \cdot \underbrace{\frac{\partial x}{\partial y}}_{=0} + \frac{\partial f}{\partial y} \cdot \underbrace{\frac{\partial y}{\partial y}}_{=1} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial y} = 0$$

$$\Rightarrow \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial y} = 0.$$

$$\Rightarrow \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial y} = -\frac{\partial f}{\partial y}$$

$$\Rightarrow \frac{\partial z}{\partial y} = \frac{-\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}} = -\frac{f_y}{f_z}$$

Ex Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(0,0,0)$ if

$$x^3 + z^3 + y e^{xz} + z \cos y = 0$$

Sol^n

$$\frac{\partial z}{\partial x} = -\frac{f_x}{f_z} = 0$$

Check: $f_z(0,0,0) \neq 0$

$$= \quad \frac{\partial z}{\partial x} = -\frac{f_x}{f_z} = 0 \qquad f_z(0,0,0) \ldots$$

$$\& \quad \frac{\partial z}{\partial y} = -\frac{f_y}{f_z} = -1$$

$f_x = 3x^2 + zy e^{xz}$

$f_y = e^{xz} - z \sin y$

$f_z = 3z^2 + xy e^{xz} + \underline{\cos y}$

$f_z(0,0,0) = 1. \neq 0$

	Function	Domain	Range.
a)	$z = \sqrt{y - x^2}$	$y \geq x^2$	$[0, \infty)$
b)	$z = \frac{1}{xy}$	$xy \neq 0$	$(-\infty, 0) \cup (0, \infty)$
c)	$z = \sin xy$	Entire plane	$[-1, 1]$