Sequence and series continue...

Thursday, October 8, 2020 6:08 AM

H/W

1. B. 2. e, f.

e. 2, 3, 6, 18, 108, 1944 ...

5th

$$\boxed{u_n = u_{n-1} \times u_{n-2}}$$

$$\left\{ \begin{array}{l} u_1 = 2 \\ u_2 = 3 \end{array} \right\}$$

set $n=5$ →

$$u_5 = u_{5-1} \times u_{5-2}$$

$$= u_4 \times u_3$$

$$= 18 \times 6 = 108$$

f) 1, 2, 6, 24, ...

1×1, 1×2, 2×3, 6×4 24×5

1st, 2nd, 3rd, 4th   5th

$$u_n = u_{n-1} \times n$$

Series:

A series is created when the terms of a sequence are added together.

$$\boxed{1 + 4 + 7 + 10 + 13}$$ ← series.

5th.

$u_n = 3n - 2$

Sigma: Σ add the numbers

$$\sum^{5} (3n-2) = 1 + 4 + 7 + 10 + 13$$

$$\sum_{n=1}^{5} (3n-2) = \underline{1} + \underline{4} + \underline{7} + \underline{10} + \underline{13}$$
$$= 35$$

Ex # $\displaystyle\sum_{n=1}^{4} (-1)^n n^2 = \underline{\underline{10}}$

write series in sigma notation.

a) $-3+5+13+21+29$

general term:- $8n-11$

$$\sum_{n=1}^{5} 8n-11$$

Arithmetic sequence:- (constant difference)

$\boxed{u_1}$, $\boxed{u_1+d}$, u_1+d+d , $u_1+d+d+d$, ---

$\underset{1^{st}}{\underline{u_1}}$, $\underset{2^{nd}}{\underline{u_1+d}}$, $\underset{3^{rd}}{\underline{u_1+2d}}$, $\underset{4^{th}}{\underline{u_1+3d}}$, ---

$$\boxed{u_n = u_1 + (n-1)d}$$

$u_{10} = u_1 + (10-1)d$

$\underline{\underline{u_{10} = u_1 + 9d}}$

Ex $\underline{\underline{-7}}, -5, -3, ----$ find u_{21}

$d = -5 - (-7) = 2$ ~~25~~ ~~35~~

$u_n = u_1 + (n-1)d$

$u_{21} = u_1 + 20d$

$= -7 + 20(2)$

$= -7 + 40 = 33$

Ex Given an arithmetic sequence in which $u_1 = 14$ and $d = -3$, find the value of n. Such that $u_n = 2$

Soln $u_1 = 14, \quad d = -3$

$u_n = 2$

Since $u_n = u_1 + (n-1)d$

$2 = 14 + (n-1)(-3)$

$2 = 14 + (-3n + 3)$

$2 = 14 - 3n + 3$

$3n = 17 - 2 = 15$

$\boxed{n = 5}$

Ex Two terms in an arithmetic sequence are $u_6 = 4$ & $u_{11} = 34$, Find u_{15}

req. u_1, d

$u_n = u_1 + (n-1)d$

$u_6 = 4$

$u_1 + (6-1)d = 4$

$u_{11} = 34$

$u_1 + (11-1)d = 34$

$u_1 + (6-1)d = 4$ | $u_1 + (11-1)d = 34$

$u_1 + 5d = 4$ — (1) $\qquad$ $u_1 + 10d = 34$ — (2)

(2) − (1)

$$\begin{array}{r} \cancel{u_1} + 10d = 34 \\ -\cancel{u_1} + 5d = 4 \\ \hline 5d = 30 \end{array}$$

$\boxed{d = 6}$ plug it in (1)

$u_1 + 5(6) = 4$

$u_1 = 4 - 30 = -26$

$\boxed{u_1 = -26}$

$u_{15} = -26 + (15-1)(6)$

$= -26 + 14 \times 6$

$= 58$

Geometric Sequence: constant ratio (r).

$u_1, \; r \cdot u_1, \; r \times r \times u_1, \; \ldots$

$\qquad 2^{nd} \qquad 3^{rd}$

$\boxed{u_n = u_1 r^{n-1}}$ (*)

EX $\quad 40, 20, 10, \ldots \; ; \; u_{12}$

$u_1 = 40$

$r = \frac{20}{40} = \frac{10}{20} = \frac{1}{2}$

$u_{12} = u_1 \cdot r^{12-1}$

$u_{12} = u_1 \cdot r^{11}$

$\gamma = \frac{20}{40} = \frac{10}{20} = \frac{1}{2}$ $\qquad$ u_{12}

$$u_{12} = 40 \times \left(\frac{1}{2}\right)^{11}$$

$$= 40 \times \frac{1}{2048}$$

$$\boxed{u_{12} = \frac{5}{256}}$$

Ex $\quad 87,\ 8.7,\ 0.87,\ ---\quad ;\ u_6$

$$u_6 = 8.7 \times 10^{-4}$$

$$= 0.00087$$

Ex In a geometric sequence $u_1 = 5$ & $u_5 = 1280$. The last term of the sequence is 20480. Given that r is positive, find the number of terms in the sequence.

For GP.

$u_n = u_1 \cdot r^{n-1}$

Soln

$u_1 = 5$, r is positive.

$u_5 = 1280$

$$u_1 \cdot r^{5-1} = 1280.$$

plug $u_1 = 5$

$$5 \cdot r^4 = 1280$$

$$r^4 = \frac{1280}{5} = 256$$

$r = 4$

$u_n = u_1 \cdot r^{n-1}$

$20480 = 5 \times (4)^{n-1}$

$n = 7$

$4096 = 4^{(n-1)}$

$4^6 = 4^{n-1}$

$6 = n-1$

$n = 7$

Application:

Ex The value of a car depreciates at a rate of 19% each year. If you buy a new car today for \$33560, how much will it be worth after four years?

19%

Ans = \$ 14446.48

$r = 81\% = 0.81$

$u_n = u_1 r^{n-1}$

$u_5 = u_1 (0.81)^{5-1}$

$1 + 2 + 3 + \dots + 98 + 99 + 100.$

$$\left\{ \begin{matrix} 101 \\ 101 \\ 101, \dots \end{matrix} \right\}$$

there are 50 pairs

n

n

there are 50 pairs

$\frac{100}{2} \times (1 + 100)$ $\boxed{50} \times 101 = 5050$

General $\rightarrow$ $\boxed{S_n = \frac{n}{2}(u_1 + u_n)}$

$\boxed{u_1, d}$

AP $\downarrow$

$u_n = u_1 + (n-1)d$

$S_n = \frac{n}{2}(u_1 + u_1 + (n-1)d)$

$$\boxed{S_n = \frac{n}{2}(2u_1 + (n-1)d)}$$

Sum of n terms of Arithmetic Series.

$u_1 = 5$, $d = 2$

$n = 10$

$S_{10} = \frac{10}{2}(2 \times 5 + (10-1)2)$

$= 5(10 + 18)$

$$\boxed{S_{10} = 140}$$