Continuous random variable

Monday, January 11, 2021 4:56 PM

Ex:

X is a random variable

$E(X) = 17.2$, $Var(X) = 45$

$n = ?$

Find the smallest sample size for which the standard deviation of the sample mean may not be greater than 2.

Soln:

$Var(X) = \sigma^2 = 45$

$E(\bar{X}) = \mu$

$Var(\bar{X}) = \frac{\sigma^2}{n}$

$S.d = 2$, $Var = 2^2$

$Var(\bar{X}) \le 2^2$

$\frac{\sigma^2}{n} = \frac{45}{n} \le 2^2$

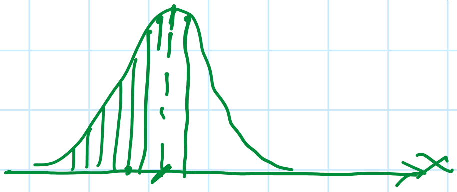
$\Rightarrow \frac{45}{4} \le n$

$\Rightarrow n \ge 11.25$

$\approx \underline{n = 12}$ this is the smallest sample size that meet the given criteria.

\# Discrete random variable $\sum p = 1$ (PMF) probability mass function

\# Continuous random variable: $\int p \, dx = 1$. (Pdf) probability density fn.

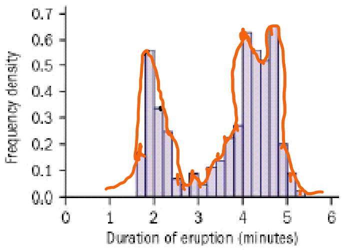


$P(w \ge 50 \text{ kg})$

$P(w \ge 50.5 \text{ kg})$

2. The histogram shows the durations of the eruptions of the Old Faithful Geyser in Yellowstone Park over a period of time.

Sketch what you think the graph of the continuous random variable looks like.



Probabilty density function:-

1) $f(x) \geq 0$

2) $\int_{-\infty}^{+\infty} f(x)\,dx = 1.$

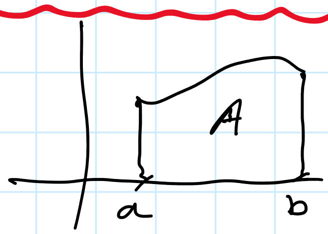
probabilty is Area under the curve.

Example 1

Show that $f(x)$ is a probability density function where

$f(x) = \frac{1}{2}(x-3)$ for $3 \leq x \leq 5$.

Find $P(X < 4)$.



area under the curve.

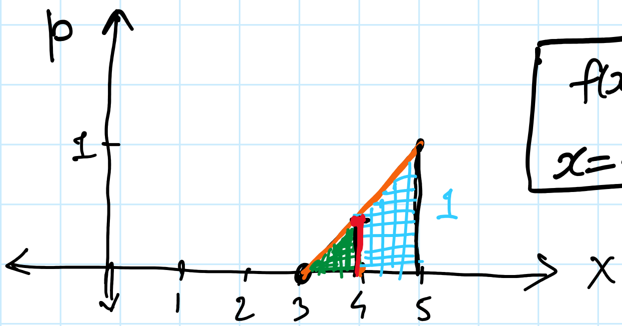
$x=3, \quad f(x) = 0$

$x=5, \quad f(5) = 1$

$$\int_3^5 \frac{1}{2}(x-3)\,dx = \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^5$$

$$= \frac{1}{2}\left\{\frac{25}{2} - 15 - \frac{9}{2} + 9\right\}$$

$$= \frac{1}{2}\left\{8-6\right\} = 1.$$



$f(x) = \frac{1}{2}(x-3)$

$x=4, \quad f(4) = 1/2$

$A = \frac{1}{2} \times 2 \times 1 = 1$

(pdf)

$P(X<4) = \int_3^4 \frac{1}{2}(x-3)\,dx = \qquad = \frac{1}{2} \times 1 \times \frac{1}{2} = \frac{1}{4}$

$$3$$

$$= \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^4$$

$$= \frac{1}{2}\left[\frac{16}{2} - 12 - \frac{9}{2} + 9\right]$$

$$= \frac{1}{2}\left[\frac{7}{2} - 3\right] = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

In continuous random variable

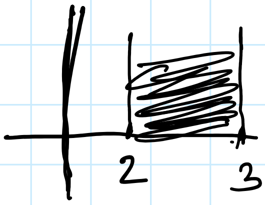
$$\left\{ \begin{matrix} \leq \\ \geq \end{matrix} \right\} \sim \left\{ \begin{matrix} < \\ > \end{matrix} \right\}$$

$(X \geq 2)$

$(X > 2)$

$$\int_2^\infty$$

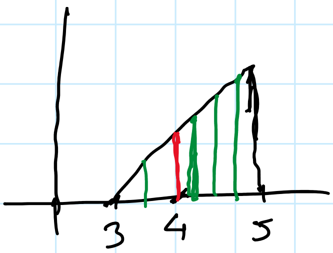
$P(X=2) = 0$



$$\int_2^3$$

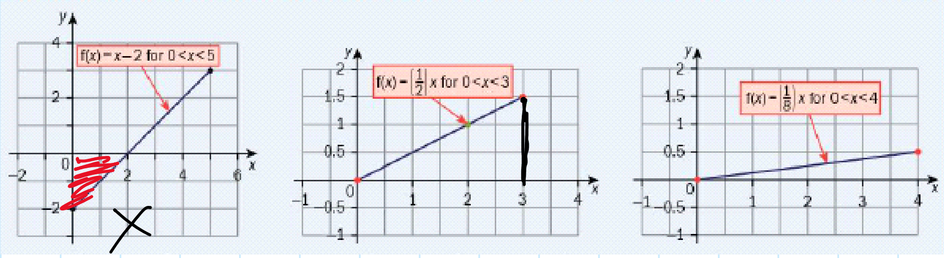
$P(X=4) = ?$

$P(X=4) = 0$



area of line = 0

Ex For the following functions, state whether they could be used as a pdf, and if not **explain** why.



1) $f(x) \geq 0$

2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Ex $f(x) = k(9 - x^2)$ for $-3 \leq x \leq 3$

Find a) the value of k and calculate $P(-1 < X < 2)$

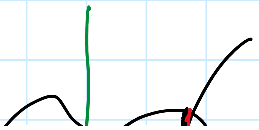
b) $\boxed{P(X = 2)}$.

For continuous distribution the probability at single value is 0 (Area of line is 0)

$f(x) = k(9 - x^2)$ for $-3 \leq x \leq 3$

pdf

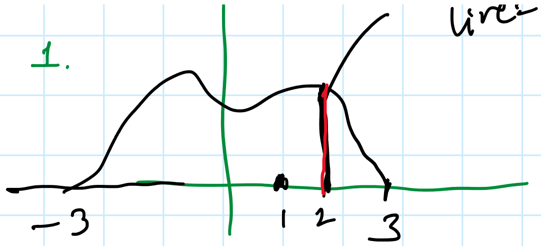
$$\int^3 f(x)\,dx = 1.$$



pdf

$$\int_{-3}^{3} f(x)\,dx = 1.$$

$3x - 2k^{1/2}$



$$\Rightarrow \int_{-3}^{3} k(9-x^2)\,dx = k\int_{-3}^{3} 9 - x^2\,dx = 1 \quad \text{(pdf)}$$

$$= k\left[9x - \frac{x^3}{3}\right]_{-3}^{3}$$

$$= k\left\{27 - \frac{27}{3} - \left(-27 - \frac{-27}{3}\right)\right\}$$

$$= k\{27 - 9 + 27 - 9\}$$

$$= k\{54 - 18\}$$

(pdf)

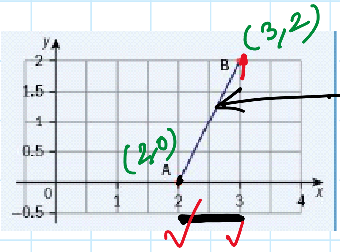
$$1 = 36k$$

$$\boxed{k = \frac{1}{36}}$$

b) $P(x=2)=0$

Ex

Express the pdf shown in this sketch as a function.



line

eqn of lines-

$$y - y_1 = m(x - x_1)$$

$$y = \left(\frac{2-0}{3-2}\right)(x-2)$$

$$\boxed{y = 2x - 4}$$

$f(x) = 2x - 4, \quad 2 \leq x \leq 3$

Median: of a data set is a value which has half the data less than or equal to it.

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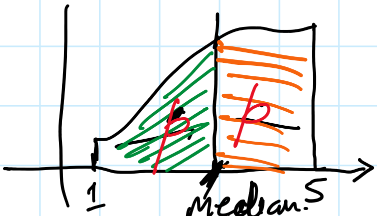
(α)



$p = 0.5$

$$M = \int_{-\infty}^{\alpha} f(x)\,dx = 0.5$$

Ex

Find the median for the probability density function in Example 1:
$f(x) = \frac{1}{2}(x-3)$ for $3 \le x \le 5$.

Median: α

$$\int_3^{\alpha} f(x)\,dx = 0.5$$

$$\int_3^{\alpha} \frac{1}{2}(x-3)\,dx = 0.5$$

$$\frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^{\alpha} = \frac{1}{2}$$

$$\left[\frac{\alpha^2}{2} - 3\alpha - \left(\frac{9}{2} - 9\right)\right] = 1$$

$$\frac{\alpha^2}{2} - 3\alpha - \frac{9}{2} + 9 = 1$$

$$\alpha^2 - 6\alpha - 9 + 18 = 2$$

$$\boxed{\alpha^2 - 6\alpha + 7 = 0}$$

$$\alpha = 3 + \sqrt{2}$$

$$\left[\frac{1}{4}x^2 - \frac{3}{2}x\right]_3^{\alpha} = 0.5$$

$$\left(\frac{\alpha^2}{4} - \frac{3\alpha}{2}\right) - \left(\frac{9}{4} - \frac{9}{2}\right) = 0.5$$

$$\frac{\alpha^2}{4} - \frac{3\alpha}{2} - \frac{9}{4} + \frac{9}{2} = \frac{1}{2}$$

$$\alpha^2 - 6\alpha - 9 + 18 = 2$$

$$\alpha^2 - 6\alpha + 7 = 0$$