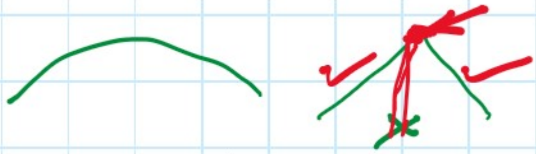
Smoothness of curve

Sunday, July 5, 2020 4:06 PM



$$u_n : \begin{cases} u_1 = 1 \\ u_{n+1} = 1 + \dfrac{1}{u_n} \end{cases} \qquad n \in \mathbb{Z}^+$$

$u_n \to L$, $u_{n+1} \to L$

$$\lim_{n\to\infty} u_n = \lim_{n \to 0} (u_{n+1})$$

$$= \lim_{n\to\infty}\left(1 + \frac{1}{u_n}\right)$$

$$\Rightarrow \quad L = 1 + \frac{1}{\lim\limits_{n\to\infty} u_n}$$

$$\Rightarrow \quad L = 1 + \frac{1}{L}$$

$$\Rightarrow \quad L^2 - L - 1 = 0$$

$$\Rightarrow \quad L = \frac{1 \pm \sqrt{5}}{2}$$

$$L = \frac{1+\sqrt{5}}{2}$$

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$\Downarrow$

$$b^2 - 4ac = (-1)^2 - 4(1)(-1)$$

$$= 1 + 4 = \sqrt{5}$$

$$\left(\frac{0}{0}\right), \quad \frac{\infty}{\infty}, \quad \pm\infty \pm \infty, \quad \infty^n$$

$$\pm\infty \times (\pm\infty),$$

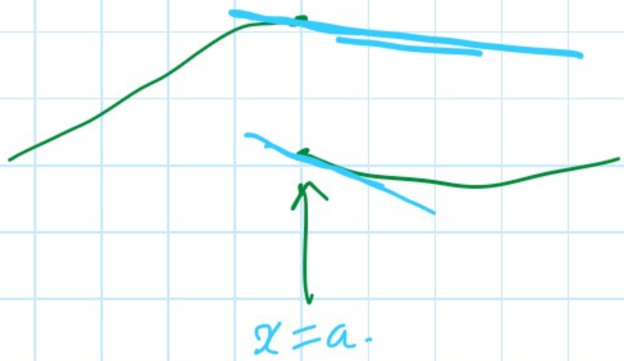
Continuity & Differentiability.

\# All differentiable f^n are continuous or Not ?

Yes

Not ?

Yes



\# $f(x)=|x|$

It is not neccessary for all continuous function to be differentiable.

Ex $\displaystyle\lim_{n\to\infty} \frac{4n^2+1}{3n^2-1} = \frac{4}{3}$

Ex $\displaystyle\lim_{n\to\infty} \sqrt{n+1}-\sqrt{n} = \lim_{n\to\infty} \frac{(\sqrt{n+1}-\sqrt{n})(\sqrt{n+1}+\sqrt{n})}{(\sqrt{n+1}+\sqrt{n})}$

$\displaystyle= \lim_{n\to\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{\infty} = \underline{0}$

Limit of Sequence to Limit of functions-

\# $f: I \to R$, $\displaystyle\lim_{x\to a} f(x)$ exists

$\displaystyle\lim_{x\to a} f(x) = b.$ if and only if

for any sequence. $a_n \in I.$ $\forall\, n \in Z^+$

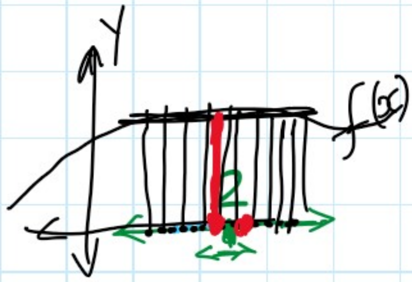
$\boxed{\lim a_n = a}$

$$\boxed{\lim_{n\to\infty} a_n = a},$$

$$\lim_{n\to\infty} f(a_n) = b$$

Ex f $f(x) = \begin{cases} 3x-1, & x < 2 \\ x-2, & x \geq 2 \end{cases}$ has no

Limit at $x = 2$



Sol^n

$U_n = 2 + \frac{1}{n}$ & $V_n = 2 - \frac{1}{n}$

$$\lim_{n\to\infty} u_n = 2 = \lim_{n\to\infty} V_n$$

if $\lim_{n\to\infty} f(u_n) = \lim_{n\to\infty} f(V_n)$ the limit exist.

* $\lim_{n\to\infty} f\left(2+\frac{1}{n}\right) = \lim_{n\to\infty} 2 + \frac{1}{n} - 2 = 0$

** $\lim_{n\to\infty} f\left(2-\frac{1}{n}\right) = \lim_{n\to\infty} 3\left(2-\frac{1}{n}\right) - 1 = 5$

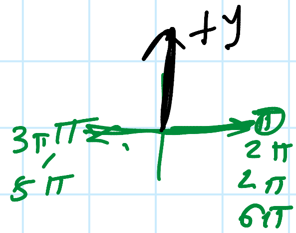
Ex! $f(x) = \sin x$, show that $\lim_{x\to+\infty} f(x)$ doesnot exist.

Sol^n

$u_n = \pi n$ $V_n = 2\pi n + \frac{\pi}{2}$

\# $\lim_{n\to+\infty} \pi n = +\infty = \lim_{n\to+\infty} 2\pi n + \frac{\pi}{2}$.

$\lim_{n\to+\infty} \pi n = +\infty \quad \lim_{n\to+\infty} \quad 2.$

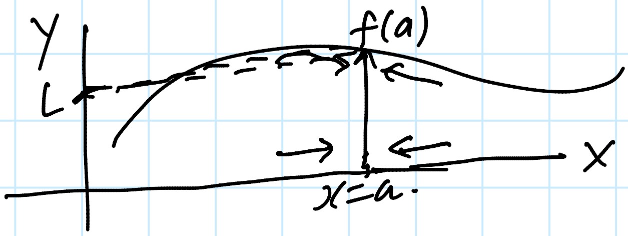


$\lim_{n\to\infty} \sin \pi n = \lim_{n\to\infty} 0 = 0$

$\lim_{n\to\infty} \sin\left(2\pi n+\frac{\pi}{2}\right) = \lim_{n\to\infty} (1) = 1.$

$$\lim_{n\to+\infty} f(u_n) \neq \lim_{n\to+\infty} f(v_n)$$

$\lim_{n\to+\infty} f(x)$ does not exist.



$$LHL = \lim_{x\to a^-} f(x) = L$$

$$RHL = \lim_{x\to a^+} f(x) = L$$

$$x=a \longrightarrow \quad f(a) = L.$$

Must be equal.

function is continuous.

Ex: Discuss the continuity. If f is discontinuous remove it discontinuity.

$$f(x) = \begin{cases} x^2, & x\neq 2 \ \left(\begin{matrix} x>2 \\ x<2 \end{matrix}\right) \\ \cancel{6}\ 4, & x=2 \end{cases}$$

$f(2) = 6$

$LHL = \lim_{x\to 2^-} x^2 = 4$

$$LHL = \lim_{x \to 2^-} x^2 = 4$$

$RHL = 4.$

Differentiable f^n:-

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ 1^{st}

at $x=a$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Ex $f(x) = \sin x$. , $f'(x) = ?$

$$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x(\cos h - 1)}{h} + \frac{\sin h \cos x}{h}$$

$$= \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \sin x \cdot (0) + \cos x \cdot (1)$$

$$= \cos x.$$

Differentiability & continuity:-