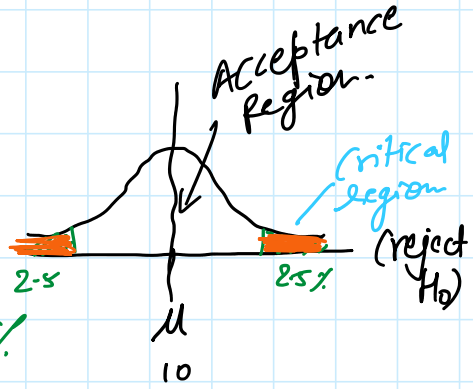
Hypothesis test for the mean of normal distribution

Thursday, April 15, 2021 4:58 PM

Null Hypothesis :- H_0

Alternative Hypothesis :- H_1.



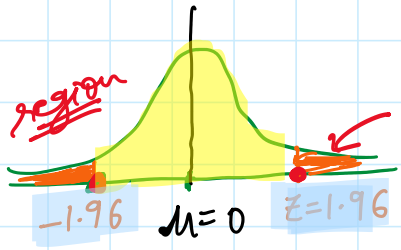
- we find critical region.

Type I. : Rejecting H_0.

Type II: Accept H_0 wrongly [conditional probability]

[Standardized normal distribution (z-distribution]

actual parameter



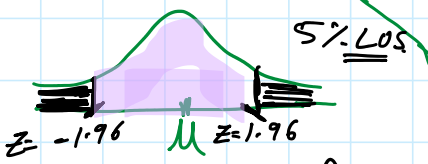
$\sigma = 1.$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

sample mean

actual mean.

LOS: 5% , $Z = 1.96$



Accept H_0 at 5% level of significance if

$$* \quad \bar{x} \in \left(\mu - 1.96 \times \frac{\sigma}{\sqrt{n}} , \mu + 1.96 \times \frac{\sigma}{\sqrt{n}} \right)$$

$$1.96 = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\bar{x} = \mu + 1.96 \frac{\sigma}{\sqrt{n}}$$

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$N(20.0, 0.16) \equiv (\mu, \sigma^2)$ ← variance

Standard deviation = 0.4

$\mu = 20.0$, $\sigma^2 = 0.16$ ← parameter:

(mu) (Sigma)

Null Hypothesis H_0 :- $\mu = 20.0$

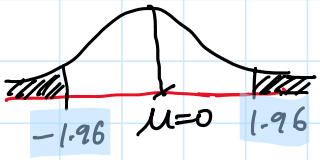
5% LOS

(mu) (sigma)

Null Hypothesis $H_0 :- \mu = 20.0$

Alternative Hypothesis $H_1 :- \mu \neq 20.0$

5% LOS



$n = 10$, $\bar{x} = 19.8$

method 1: $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{19.8 - 20.0}{\frac{0.4}{\sqrt{10}}} = -1.5811$

this lies in acceptance region (between -1.96 to 1.96)

Accept H_0.

method: 2. $\bar{x} = 19.8$

$$\left(20.0 - 1.96 \times \frac{0.4}{\sqrt{10}} , 20.0 + 1.96 \times \frac{0.4}{\sqrt{10}}\right)$$

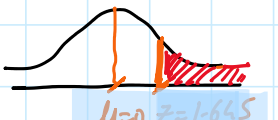
$\bar{x} \in = (19.75, 20.25)$

Accept H_0

Ex 2 $H_0 : \mu = 82.3$, $H_1 : \mu > 82.3$ one tail test.

$\sigma = 15.3$, $n = 5$

5% LOS (one tail test) $z = 1.645$

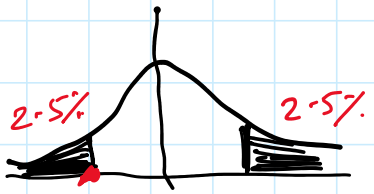


Rejection region

$$\bar{x} > \mu + z \times \frac{\sigma}{\sqrt{n}} = 82.3 + 1.645 \times \frac{15.3}{\sqrt{5}}$$

HW

$\bar{x} > 93.6$

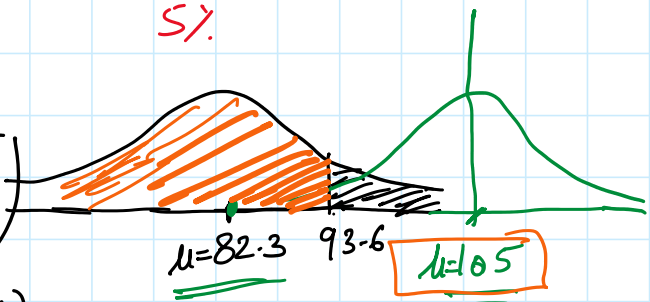


(i) $P(\text{Type I}) = 5\% = 0.05$

(ii) mean rainfall = 105

5%

$$P(\bar{x} < 93.6) = P\left(z < \frac{93.6 - 105}{\frac{15.3}{\sqrt{5}}}\right)$$



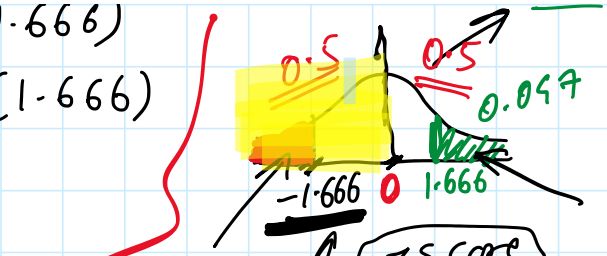
$= P(z < -1.666)$

$\Phi(1.666)$

$= P(z < -1.666)$

$= 1 - \phi(1.666)$

$= 4.8\%$



Z-score

Lower limit Upper limit.

$-1.66 \longleftrightarrow 0$

$0.5 - 0.405 = 4.8\%$.