Hypothesis test for the mean of poisson distribution

Saturday, April 3, 2021 10:57 AM

1) Critical region.

2) probability approach.

Q A single observation x is to be taken from a Poisson distribution with parameter λ.
This observation is to be used to test $H_0: \lambda = 6$ against $H_1: \lambda \neq 6$.

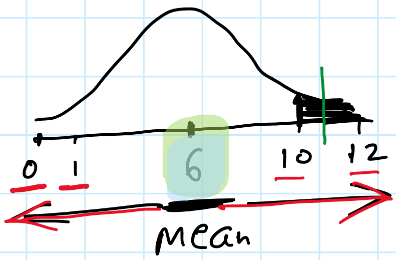
i) Using a 5% significance level, find the critical region for this test assuming that the probability of rejection in either tail is to be no more than 2.5%. 0.025

ii) Write down the actual significance level of this test.

The actual value of x obtained was 2.

iii) State a conclusion that can be drawn based on this value.

$H_0: \lambda = 6$, $H_1: \lambda \neq 6$ (Two tail test)



$P(x \leq 0) = 0.0024 < 0.025$

$P(x \leq 1) = 0.0173 < 0.025$

~~$P(x \leq 2) = 0.0619 > 0.025$~~

$\boxed{x = 0, 1.}$

$= 1 - P(x \leq 9)$

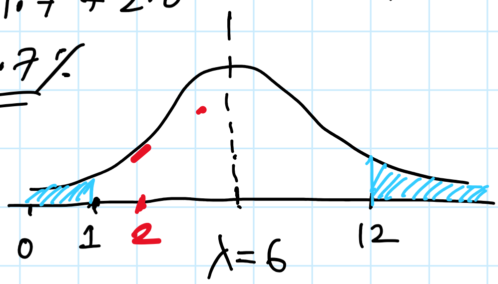
$P(x \geq 10) = 0.084 > 0.025$

$P(x \geq 11) = 1 - P(x \leq 10)$
$= 0.0427 > 0.025$

$P(x \geq 12) = 1 - P(x \leq 11)$
$= 0.0201 < 0.025$

$\boxed{x \geq 12}$

Actual level of significance $= 1.7 + 2.0$
$= 3.7\%$



(c) $\lambda = 2$ is not in the critical region
we accept H_0 and conclude that there is not significance evidence at the 5 % level of significance. to suggest that the mean is not 6.

Q The number of accidents on a stretch of road may be modelled by a Poisson distribution with an average rate of 2.5 accidents per week. $\lambda = 2.5$

a) Find the probability that there are at least 5 accidents in a randomly chosen week. $\times 3$

Some new warning notices are installed and in a 3-week period there are 4 accidents. ← sample $\lambda = 7.5$

b) Using a 5% significance level, test whether there has been a reduction in the average rate of occurrence of accidents.

(a) $X \sim Po(2.5)$

$P(X \geq 5) = 1 - P(X \leq 4) = 0.1088$ ✓

(b) $H_0 : \lambda = 7.5$, $H_1 : \lambda < 7.5$ (One tail test)

$\boxed{X = 4}$ ← statistics Test.

critical region. probabiliy approach.

$P(X \leq 2) = 0.0203$, ~~$P(X \leq 3) = 0.059 > 5\%$~~

Actual significance level $= 2.03\%$

critical region: $0, 1, 2$

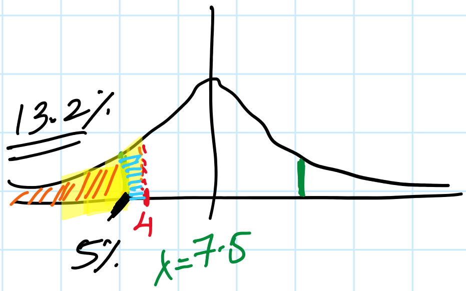
$X = 4$

since 4 does not lies in critical region, H_0 would be accepted.

H_0 would be accepted.

probability approach: $P(X \le 4) = \underline{\underline{0.132 > 0.05}}$

$X = 4$ is in Acceptance region.



Luigi was recently appointed to be responsible for the service in a restaurant. During the previous year, the restaurant received an average of 3 emails per week complaining about the quality of service in the restaurant. $\lambda = 3$

The number of such emails may be modelled by a Poisson distribution.

a) During the week before Luigi's appointment, 6 such emails were received. Examine, at the 5% level of significance, whether there is significant evidence that, immediately before Luigi's appointment, the mean number of such emails received exceeded 3 per week.

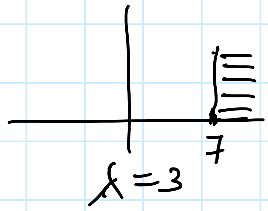
b) On his appointment, Luigi introduced changes to the methods of waiters recording orders and passing them to the kitchen. Following these changes, 2 emails of complaint were received during a two-week period. $\lambda = 6$ $Y = 2X$
Examine, using a 5% level of significance, whether there is significant evidence that, following the changes introduced by Luigi, the mean number of such emails received was less than 3 per week.

c) Comment on the effectiveness of the changes introduced by Luigi.

$H_0: \lambda = 3$, $H_1: \lambda > 3$

$P(X \ge 7) = 1 - P(X \le 6)$

$= 0.0336$



critical region

6 is in Acceptance region.

Hence H_0 is accepted.

Alternative approach:

$P(X \ge 6) = 1 - P(X \le 5)$

$= \underline{0.084 > 0.05}$

Hence $X = 6$ is not in critical region.

(b) $H_0: \lambda = 6$, $H_1: \lambda < 6$

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$p(Y \leq 2) = 0.0619 > 5\%$

H_0 is accepted.

Hence we don't have sufficient evidence that tells us the mean number of such emails received was less than 3 per week.

c) The number of complaints has gone from 6 in a week to only 2 in a two-week period, which looks as though the changes might have had an effect. However, the tests on the mean being higher than 3 per week before and being lower than 3 per week after did not provide sufficient evidence in either case, highlighting that accepting the null hypothesis in a test is a fairly weak statement – the problem is that testing in only one or two weeks makes it very difficult to identify a shift in the mean rate. If Luigi monitored the complaints over a period of two months he would have a much better evidence base on which to judge the effects of the changes.