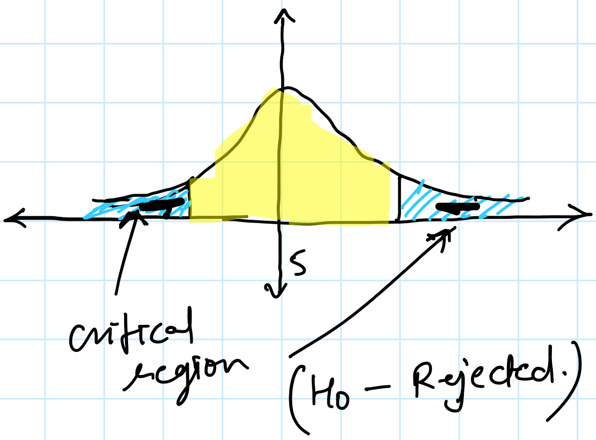
Type I and Type II error

Saturday, March 27, 2021 5:55 AM

H_0 ← Null Hyp → Accept (Failed to reject) / Reject

H_a ←

Reject
↓
Critical region.
↓ probability
Exact level of significance.



$H_0: \mu = 5$

$H_a: \mu \neq 5$ → $\mu > 5$, $\mu < 5$

LOS – 5%

Two tail test.

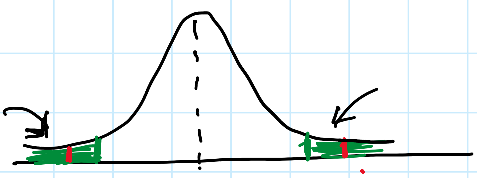
Error: What if the Null Hypothesis rejected incorrectly. (Reject Null Hypo).

Type I error:

$P(\text{Type I error}) =$ area of critical region
$=$ Exact level of significance

Type II error: Accepting Null Hypothesis incorrectly. (accept Null Hypothesis).

If I decrease the LOS from 5% to 1% (say)



1% LOS

Type I.

Type II

If we reduce the LOS to (1%) the probability of a Type I error reduce but

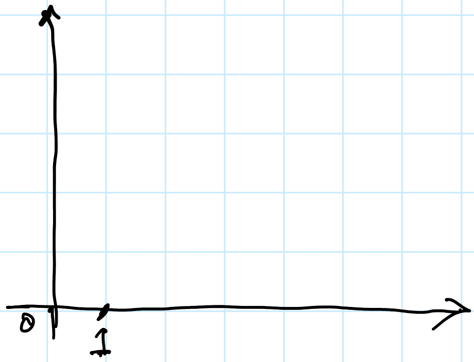
probability of a Type I error reduce but will give a higher probability of making Type II error.

\# . LOS (more than 5%) then the type I error increases.

Hence we generally take LOS = 5%

because it balances both the error together.

Ex: 8.3



1) $n = 15$, $H_0: p = 0.2$

0, 1 critical region.

$P(X \leq 1) = 0.167$

Significance level = 16.7%

(ii) $P(\text{Type I error}) = 16.7\% = 0.167$

(iii) $p = 0.1$

$P(X > 1 \mid p = 0.1) = 45.09\%$

$= 1 - P(X = 0, 1)$

$= 1 - 0.5490$

$P(\text{Type II error}) = 0.451$

(2)

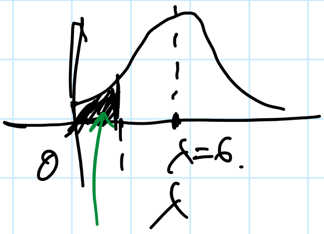
2. A test is being carried out on a Poisson distribution $H_0: \lambda = 6$ against $H_1: \lambda < 6$. A 2% test is to be carried out.
   i) Find the critical region for the test.
   ii) Calculate the actual significance level of the test.
   iii) State the probability that a Type I error occurs.
   iv) If in fact $\lambda = 3$, calculate the probability of a Type II error.

$H_0: \lambda = 6$
$H_1: \lambda < 6.$
$LOS = 2\%$

[poisson / binomial
Discrete
random
Variable]

$P(X=0) = 0.00248 < 0.02$
$P(X \le 1) = 0.0173 < 0.02$
$P(X \le 2) = 0.0619 > 0.02$

Critical region $\rightarrow X = (0, 1)$



(ii) Actual level of significance.
$= 1.7\%$

0.0173

(iii) $P(\text{Type I error}) = 0.0173$

(iv) $P(\text{Type II error} / \lambda = 3) = P(X > 1 \mid \lambda = 3)$
$= 1 - P(X \le 1 \mid \lambda = 3)$
$= 0.801 = 80.1\%$