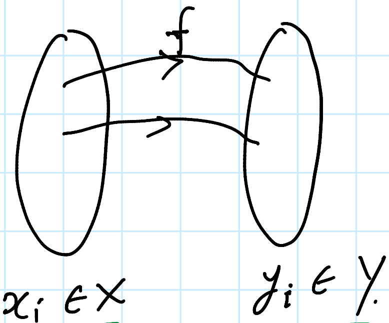
Inverse function, Linear and quadratic function

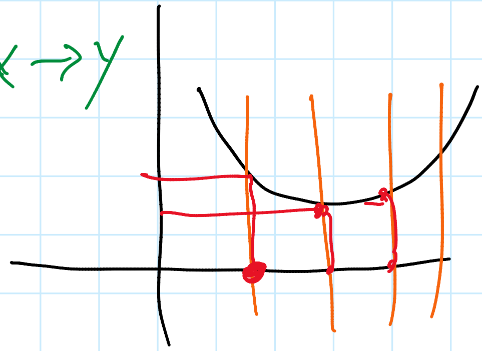
Thursday, October 29, 2020 5:43 AM



$x_i \in X$ $\quad$ $y_i \in Y$

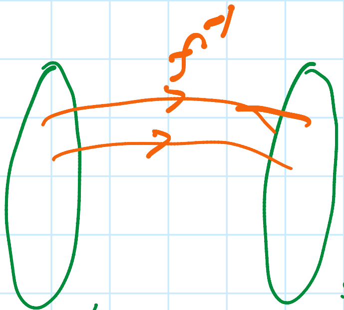
Domain $\quad$ Range.

$X \to Y$



f^{-1}

$Y \to X$

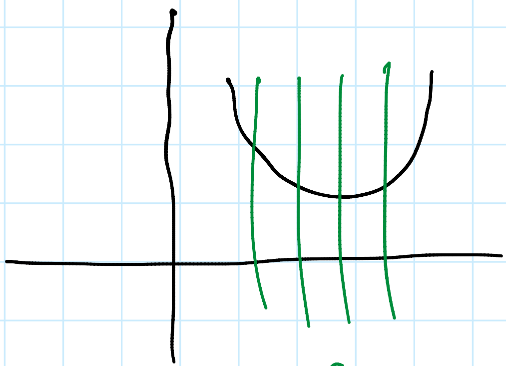


— Vertical line Test.

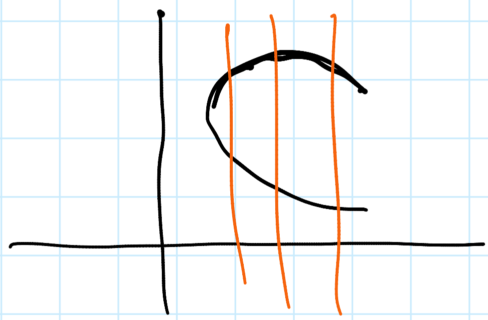
$y_i \in Y$ $\quad$ $x_i \in X$

Domain. $\quad$ Range.

1. vertical line Test
2. Horizontal line Test.

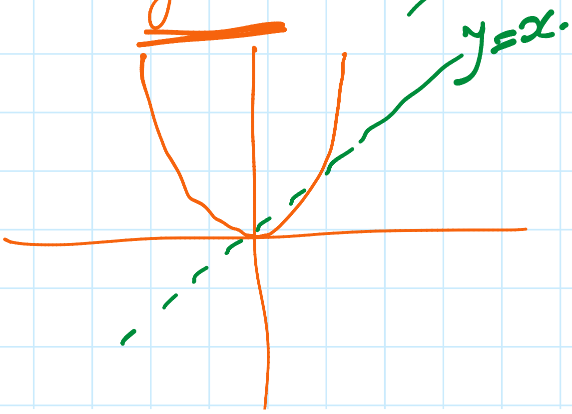


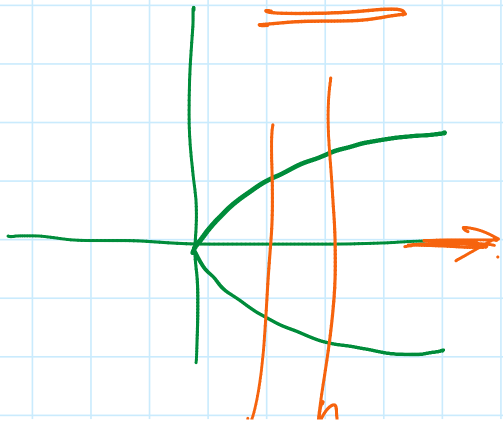
f

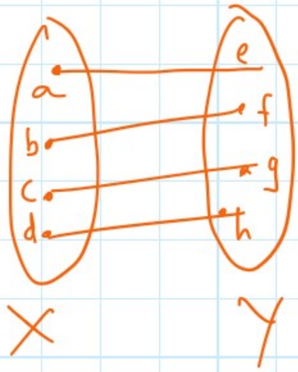


$\times f$

$y = x^2 \implies x = \pm\sqrt{y} \Longleftarrow x^2 = y.$







one-to-one.

Def:

$$\begin{cases} (f\circ g)(x) = x \Rightarrow g(x) = f^{-1}(x) \\ (g\circ f)(x) = x \Rightarrow f(x) = g^{-1}(x) \end{cases}$$

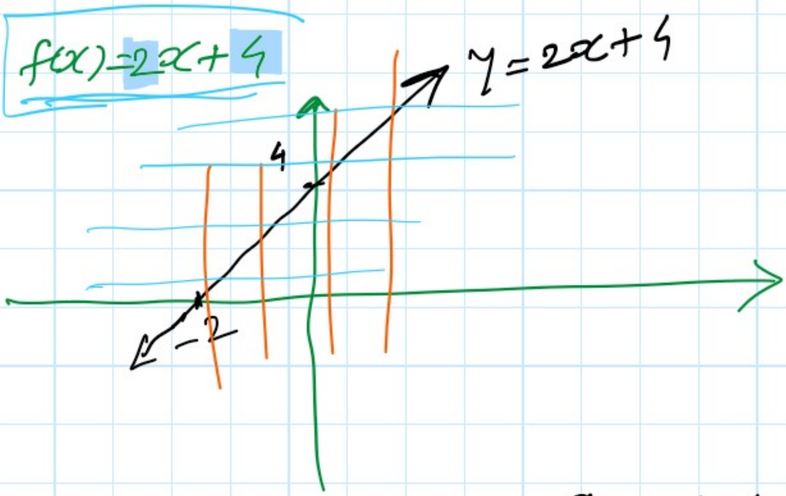
Ex: $f(x) = 2x+4$, $\boxed{g(x) = \frac{x-4}{2}}$

$\checkmark\ (f\circ g)(x) = f(g(x)) = f\left(\frac{x-4}{2}\right) = 2\left(\frac{x-4}{2}\right)+4$
$= x.$

$\checkmark\ (g\circ f)(x) = g(f(x)) = g(2x+4) = \frac{2x+4-4}{2} = x.$

$(f\circ g)(x) = x. = (g\circ f)(x)$

$f(x) = 2x+4$



$y = f(x) = 2x + 4.$

$y = 2x + 4$

$\xrightarrow{x \Leftrightarrow y}$ $x = 2y + 4$

$\longrightarrow$ $2y = x - 4$

$y = \dfrac{x-4}{2}$

$\longrightarrow$ $\boxed{f^{-1}(x) = \dfrac{x-4}{2}}$

Steps to find inverse f^n

1) Interchange x & y
2) make y as a subject
3) replace y by $f^{-1}(x)$.

Ex 1) Determine if each f^n given is one-to-one f^n

a) If it is one-to-one find inverse.

a) $f(x) = 4x + 2$ (linear f^n) is always one-to-one f^n

$f^{-1}(x) = x - \dfrac{2}{4}$

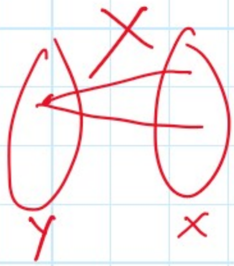
b) $f : x \rightarrow x^2 + 1.$

$y = x^2 + 1.$

$\cancel{A}$

No inverse.

GDC



c) $g(x) = \sqrt{x+2} - 3$

$y = \sqrt{x+2} - 3$

GCD.

$y = \sqrt{x}$

$\downarrow$

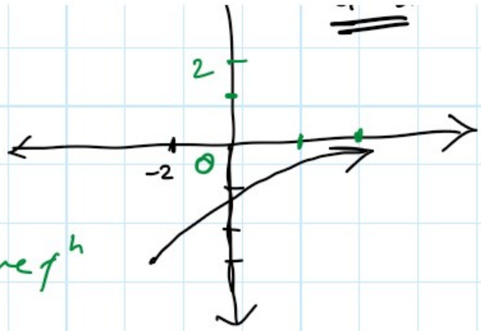
$y = \sqrt{x+2} - 3$

It is f^n.

It is one-to-one f^n

$\downarrow$

Inverse exist



$\downarrow$

$y = \sqrt{x+2}$ $\quad \begin{pmatrix} x+2=0 \\ x=-2 \end{pmatrix}$

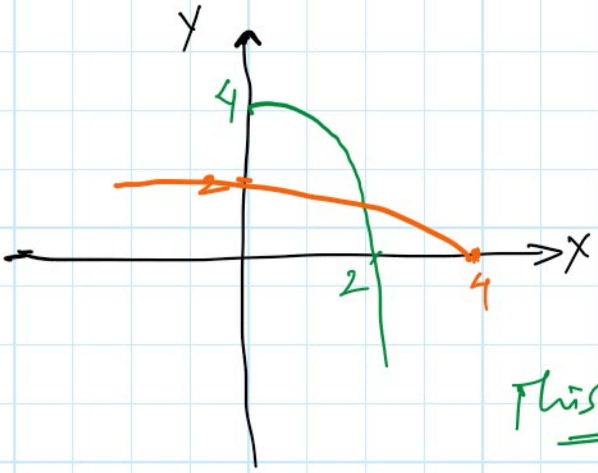
$\downarrow$

$y = \sqrt{x+2} - 3$ $\downarrow$

Find inverse f^n.

$$g^{-1}(x) = (x+3)^2 - 2.$$

Ex $\quad f : x \to -x^2 + 4$, for $x \geq 0$



$y = x^2$

$\downarrow$

$y = -x^2$

$\downarrow$

$y = -x^2 + 4.$

This is f^n.

One-one f^n (passes Horizontal & vertical Test)

Inverse exist

$f : x \to -x^2 + 4$

$y = -x^2 + 4$ $\quad \boxed{x \geq 0}$

$x \leftrightarrow y$ $\quad x = -y^2 + 4$

$y^2 = 4 - x$

$y = \pm\sqrt{4-x}$

$f^{-1}(x) = \sqrt{4-x}$

$y = -x^2 + 4$

$x^2 = 4 - y$

$x = \pm\sqrt{4-y}$

$x = \sqrt{4-y}$ $\quad (\because x \geq 0)$

$$f^{-1}(x) = \sqrt{4-x}$$

$x = \sqrt{4-y}$ $(\because x \le 0)$

$x \longleftrightarrow y$ $y = \sqrt{4-x}$

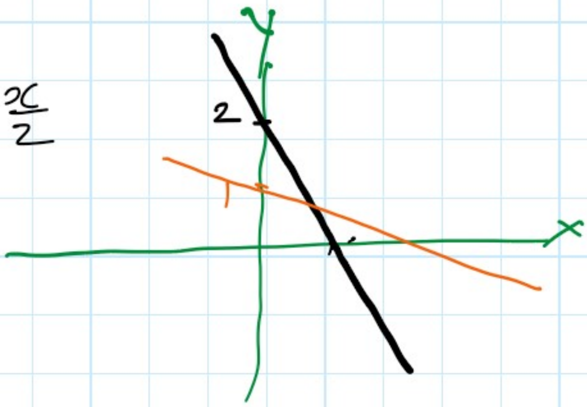
$$f^{-1}(x) = \sqrt{4-x}$$

\# function & inverse f^n are mirror image of each other about $y=x$ line.

Ex $\checkmark f(x) = -2x+2$ (linear).

$\checkmark f^{-1}(x) = \frac{2-x}{2} = 1 - \frac{x}{2}$

$\boxed{y=x}$



Ex Show $f(x) = \frac{x}{x-1}$ is self-inverse f^n.

$(f \circ f)(x) = x$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{x}{x-1}\right)$$

$$= \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1}$$

$$= \frac{\frac{x}{\cancel{x-1}}}{\frac{\cancel{x} - \cancel{x} + 1}{\cancel{x-1}}}$$

$(f \circ f)(x) = x$

$$(f \circ f)(x) = x$$

Hence f is self inverse f^n.

Linear f^n

$$y = mx + b$$

Gradient — Y-intercept.

Ex

$$\overset{x_1, y_1}{(4, 2)}, \quad \overset{x_2, y_2}{(6, 5)}$$

$$\text{Gradient} = \frac{3}{2}$$

$$\boxed{y = \frac{3}{2}x - 4} \quad \Big\} \text{ gradient-intercept form.}$$

\# point-gradient form:- $y - y_1 = m(x - x_1)$.

\# standard form.

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - 2 = \left(\frac{5-2}{6-4}\right)(x - 4)$$

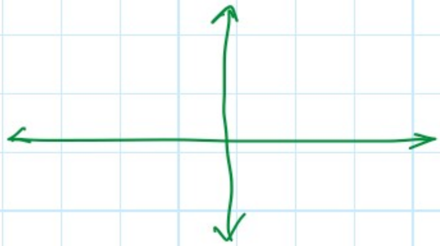
$$y - 2 = \frac{3}{2}(x - 4)$$

$$2y - 4 = 3x - 12$$

$$2y = 3x - 8$$

$$\boxed{y = \frac{3}{2}x - 4}$$

$$m_1 = m_2$$



$m_1 m_2 = -1.$ (HW)

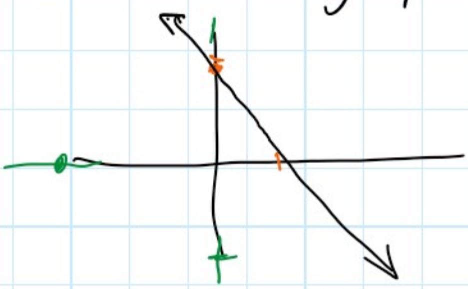
Ex find eqⁿ of line that passes through $(-4, 5)$ and is parallel to line with eqⁿ $y = -\frac{1}{2}x - 3$.

$$y = -\frac{1}{2}x + 3$$

$$y - 5 = -\frac{1}{2}(x+4)$$

$$y - 5 = -\frac{1}{2}x - 2$$

Ex Draw the graph of $y + 4 = -2(x-3)$



$$y + 4 = -2x + 6$$

$$\boxed{y = -2x + 2}$$

Ex find point of intersection of

$y = -3x + 1$ and $y = -5x + 3$

verify by GDC.

$(1, -2)$