Limit continuity-2

Wednesday, June 3, 2020 11:01 AM

Ex: $\lim_{(x,y)\to(1,0)} \frac{2xy-2y}{x^2+y^2-2x+1}$ Show D.N.E.

Along $y=0$

$\lim_{x\to 1} \frac{2x(0)-2(0)}{x^2+0^2-2x+1}$

$= \lim_{x\to 1} \frac{0}{x^2-2x+1}$

$= 0$

Along $x=1$.

$\lim_{y\to 0} \frac{2(1)y-2y}{1^2+y^2-2(1)+1}$

$= \lim_{y\to 0} \frac{2y-2y}{y^2}$

$= \lim_{y\to 0} \frac{0}{y^2} = 0$

x^2-2x+1
$= (x-1)^2$

Along C:- $y = x-1$

$\lim_{(x,y)\to(1,0)} \frac{2y(x-1)}{y^2+(x-1)^2}$

$\Rightarrow \lim_{y\to 0} \frac{2yy}{y^2+y^2} = \lim_{y\to 0} \frac{2y^2}{2y^2} = 1.$

$0 \neq 1.$

So limit DNE exist.

Ex: $\lim_{(x,y,z)\to(0,0,0)} \frac{xz^2+2y^2}{x^2+2y^2+z^4}$ (8)

Along $x=0$ $\begin{pmatrix} y=0 \\ z=0 \end{pmatrix}$

$\lim_{x\to 0} \frac{0}{x^2} = 0$

Along C!- $x = t^2$, $y = t^2$, $z = t$

$(x,y,z)\to(0,0,0)$

$t\to 0$

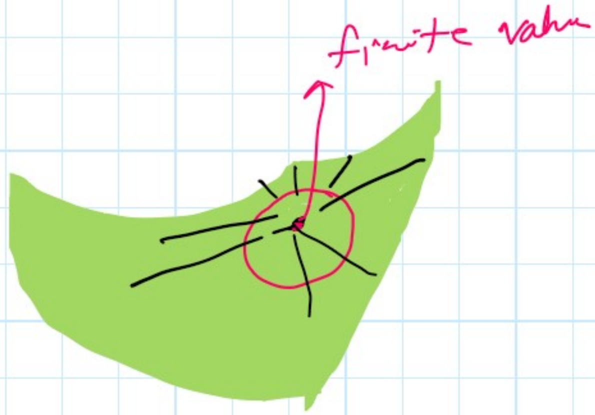
$$\lim_{t\to 0} \frac{t^2\cdot t^2 + 2(t^2)^2}{(t^2)^2 + 2(t^2)^2 + t^4} = \lim_{t\to 0} \frac{3t^4}{4t^4} = \boxed{\frac{3}{4}}$$

$0 \neq \frac{3}{4}$. Limit DNE at $(0,0,0)$.

Ex:

$$\lim_{(x,y)\to(1,-2)} \left(\frac{3xy}{2x^2 - y^2}\right)$$

$= 3$



Ex:

$$\lim_{(x,y)\to(0^+,0^+)} \left(\frac{e^{\sqrt{x+y}}}{x+y-1}\right)$$

$= -1$

Ex:

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz + xz}{x^2 + y^2 + z^2} \qquad \left(\frac{0}{0}\right)$$

$0 \neq 1$.

Limit DNE at $(0,0,0)$.

Ex:

$$\lim_{(x,y,z)\to(0,3,1)} \left[e^{\sin(\pi x)} + \ln\left(\cos(\pi(y-z))\right)\right]$$

$= 1.$

$= 1.$

Never skip plugging the value inside f^n.

Ex:
$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$$

$x = r\cos\theta$

$y = r\sin\theta$

$$\boxed{r^2 = x^2+y^2} \Rightarrow r = \sqrt{x^2+y^2}$$

$\sin^2\theta + \cos^2\theta = 1.$

$(x,y)\to(0,0)$

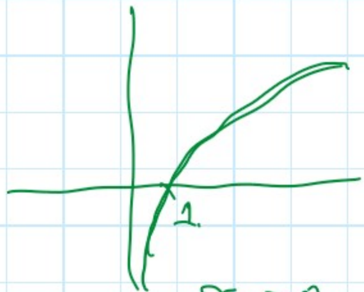
$r \to 0^+$

$$\lim_{r\to 0^+} \frac{r^3\cos^3\theta + r^3\sin^3\theta}{r^2} = \lim_{r\to 0^+} r\cos^3\theta + r\sin^3\theta$$

$$= \lim_{r\to 0^+} r(\cos^3\theta + \sin^3\theta)$$

$$= 0$$

Ex:
$$\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$$



$x \to 0$

$y \to -\infty$

$x^2+y^2 = r^2$

$(x,y)\to(0,0) \Rightarrow r \to 0^+$

$$\Rightarrow \lim_{r\to 0^+} \frac{\ln r^2}{\frac{1}{r^2}}$$

L'Hop

$$= \lim_{r\to 0} \frac{\frac{1}{\cancel{r^2}}\cdot\cancel{2r}}{\frac{-\cancel{2}}{r^3}} = \lim_{r\to 0} \frac{\frac{1}{r}}{\frac{-1}{r^3}} = \lim_{r\to 0} -\frac{r^3}{r}$$

$$= \lim -r^2 = 0.$$

$r \to 0$

Continuity:

A function is continuous at any point on the region for which it is defined. (Domain).

1) Polynomial function is continuous at all points (x, y)

2) Rational function is continuous at all points Except (Denominator $\neq 0$).

$$R(x,y) = \frac{p(x,y)}{q(x,y)}$$

3) continuity holds for composition.

$y = \sin x$. $\quad y = |\sin x|$

$y = -\sin x$.

Ex: $$f(x,y) = \frac{x^3 + xy + y^3}{x^2+y^2}$$ ✓ ✓

It is continuous at all points Except $(x,y) = (0,0)$

F^n is continuous at $\{(x,y) \mid (x,y) \neq (0,0)\}$

Ex: $$f(x,y) = \sqrt{x}\ e^{x/y}$$

F^n is continuous on $\{(x,y) \mid x \geq 0 \text{ and } y \neq 0\}$.

Ex: $f(x,y,z) = \dfrac{xyz}{x^2+y^2+z^2-4}$

continuous on $\{(x,y,z) \mid x^2+y^2+z^2 \neq 4\}$

$h(x,y) = g(f(x,y))$

Ex: $f(x,y) = x^2+xy+y^2$ $\qquad g(t) = t\cos t + \sin t$

$h(x,y) = g(f(x,y))$

f is continuous for all value.

g is also continuous at all values.

$h(x,y)$ is also continuous for all (x,y).

Ex: $f(x,y) = x-2y+3$ $\quad , \; g(x) = \sqrt{t} + \dfrac{1}{t}$

$h(x,y) = g(f(x,y))$

$t > 0$

$t = x-2y+3$

$x-2y+3 > 0$

$\boxed{x-2y > -3}$

$h(x,y)$ is continuous on $\{(x,y) \mid x-2y > -3\}$

Ex: $f(x,y) = x\tan y$ $\quad , \; g(t) = \cos t$

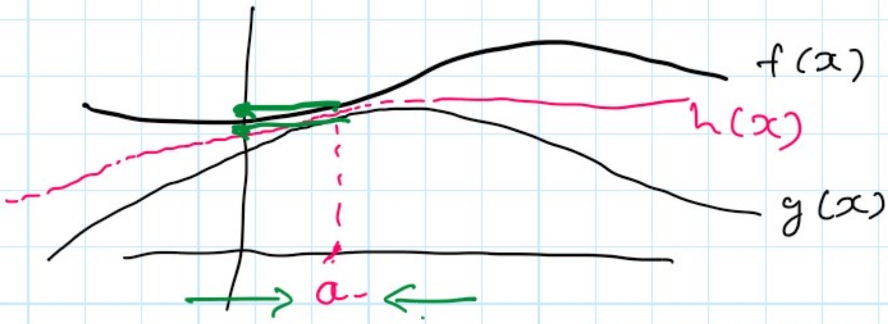
Ex: $f(x,y) = x\tan y$, $g(t) = \cos t$

f^n is continuous on $\{(x,y) \mid y \neq \frac{\pi}{2} + k\pi\}$.

continuous everywhere.

$h(x,y) = g(f(x,y))$ is continuous $\left\{(x,y) \mid y \neq \frac{\pi}{2} + k\pi\right\}$

SQUEEZE THEOREM: (SANDWICH THM)



$g(x) \leq h(x) \leq f(x)$

$\lim_{x \to a} g(x) \leq \lim_{x \to a} h(x) \leq \lim_{x \to a} f(x)$

$L \leq \lim_{x \to a} h(x) \leq L.$

$\lim_{x \to a} h(x) = L$

Ex: $\lim_{(x,y) \to (0,0)} \frac{5x^2y}{x^2+y^2}$

Soln: $5\frac{x^2}{x^2} = 5$ $\qquad y^2 \geq 0$

$\Rightarrow \frac{5x^2}{x^2+y^2} \leq 5$ $\qquad |y|$

$$\Rightarrow \quad \frac{5x^2|y|}{x^2+y^2} \le 5|y|$$

$$\Rightarrow 0 \le \left|\frac{5x^2y}{x^2+y^2}\right| \le 5|y|$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} 0 \le \lim_{(x,y)\to(0,0)} \left|\frac{5x^2y}{x^2+y^2}\right| \le \lim_{(x,y)\to(0,0)} 5|y|$$

$$\Rightarrow \quad 0 \le \lim_{(x,y)\to(0,0)} \left|\frac{5x^2y}{x^2+y^2}\right| \le 0$$

$$\lim_{(x,y)\to(0,0)} \left|\frac{5x^2y}{x^2+y^2}\right| = 0$$

The continuity holds for composition.

$$\therefore \lim_{(x,y)\to(0,0)} \frac{5x^2y}{x^2+y^2} = 0$$

DIFFERENTIALS :-

$$\begin{Bmatrix} dx & \Delta x \\ dy & \Delta y \end{Bmatrix}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

3-D.