Pattern to infinity

Saturday, July 4, 2020 5:55 PM

1) what is sequence & what is series ? [convergent/divergent]

2) $\lim_{n\to\infty}\left(\frac{\sin n}{n}\right) = ?$ $\left[\lim_{n\to 0} \frac{\sin n}{n} = 1\right]$

3) L'Hopital Rule.

$$f(a) = g(a) = 0$$

then why $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

Exercise on above topic.

a) what is sequence ?

Set of numbers that are separated by (,) comma.

i) $1, 2, 4, 8, 16, ----$

general term $a_n = 2^n$

ii) $1, \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, ------$

$\underset{(a_n)}{G.T} = \frac{1}{2^n}$

Defined order.

$\sum_{n=0}^{\infty} 2^n \leftarrow$ Series.

$2^0 + 2^1 + 2^2 + 2^3 + ----$ } Series

$a_n = \frac{1}{n}$ [Sequence]

$n \to \infty$

$\sum\left(\frac{1}{n}\right)$ Series.

$n \to \infty$

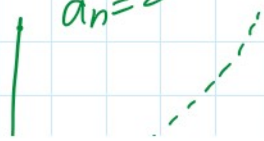
Is it convergent ?

Sequences:

a) convergent

b) divergent

$a_n = 2^n$



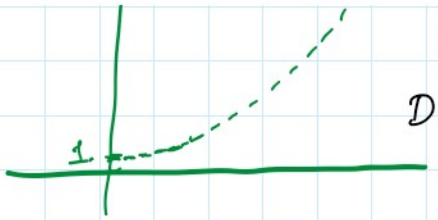
$n \to \infty$

$a_n \to \infty$

Divergent

b) divergent

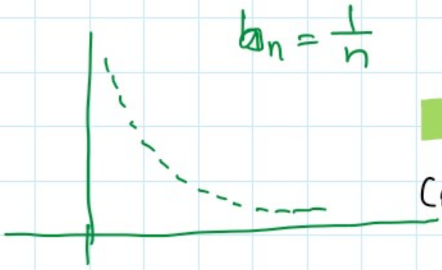
c) oscillating



$a_n \to \infty$

Divergent

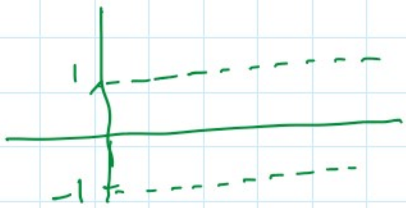
$b_n = \frac{1}{n}$



$n \to \infty$

$b_n \to 0$

Convergent



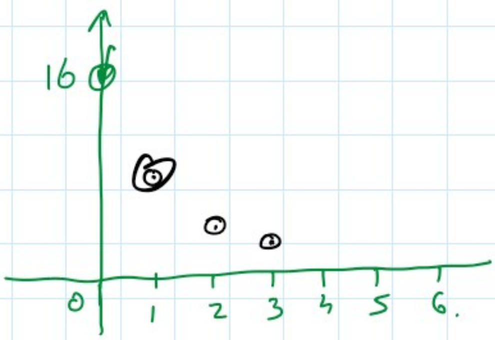
$c_n = (-1)^n$

oscillating

(Divergent)

Limit:

if $\lim_{n\to\infty} u_n = L$ then $\{u_n\}$ is convergent sequence.



$u_n = \frac{16}{2^n}$

$\lim_{n\to\infty} \frac{16}{2^n} = 0$

u_n is convergent.

$n \to 1 \quad |u_n - L| = 8$

a_n

$n \to 100 \qquad u_n = \frac{16}{2^{100}} \neq 0$

$\epsilon = 1$

m

$n \geq m$

$m = 4$

$|u_n - L| \leq \epsilon$ (epsilon)

$\epsilon = 0.00000001$

$\{u_n\}$ is a convergent sequence with $\lim_{n\to\infty} u_n = L$

if and only if for any $\epsilon > 0$ there exist a least order $m \in \mathbb{Z}^+$ such that for all

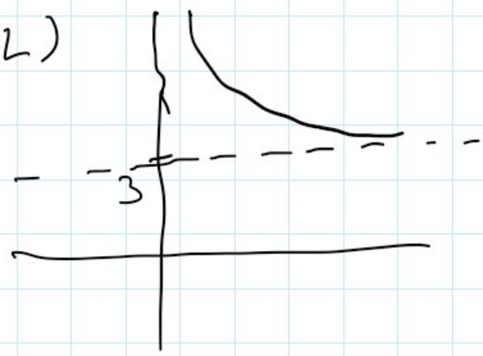
$$n \geq m \Rightarrow |u_n - L| < \epsilon \ \checkmark$$

Q Show that the sequence defined by

$$\boxed{u_n = \frac{3n-1}{n+1}}$$ is convergent.

Sol^n.

$$\lim_{n\to\infty} \frac{3n-1}{n+1} = 3\,(L)$$



$$|u_n - L| < \epsilon$$

$$\left|\frac{3n-1}{n+1} - 3\right| < \epsilon$$

$$\left|\frac{3n-1-3n-3}{n+1}\right| < \epsilon$$

$$\left|\frac{-4}{n+1}\right| < \epsilon$$

$$\boxed{\frac{4}{n+1} < \epsilon}$$

$n \geq m$

$$n+1 > \frac{4}{\epsilon}$$

$$n > \left(\frac{4}{\epsilon} - 1\right)$$

$$n > \left(\frac{4}{\epsilon} - 1\right)$$

Since $n > m$

$$m = \frac{4}{\epsilon} - 1$$

Subsequences:

\# If $\{b_n\} \subseteq \{a_n\}$

If $\{a_n\}$ is convergent $\Rightarrow \{b_n\}$ also convergent.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = L \text{ (finit value)}$$

\# if $\{b_n\} \subseteq \{a_n\}$ and $\{c_n\} \subseteq \{a_n\}$

$\lim_{n\to\infty} b_n \neq \lim_{n\to\infty} c_n$ then $\{a_n\}$ not convergent. {i.e. $\{a_n\}$ is a divergent sequence}

Ex: $a_n = (-1)^n \cdot 2$ [does not convere]

$b_n = a_{2n} = 2$ | $c_n = a_{2n-1} = -2$

$\lim_{n\to\infty} b_n = 2$ | $\lim_{n\to\infty} c_n = -2$

a_n is not convergent.

Squeeze theorem and the algebra of limit of convergent sequence:-

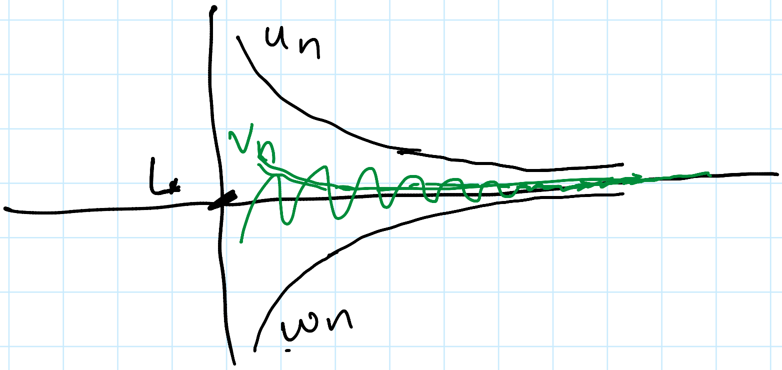
$$\lim_{n\to\infty}\left(\frac{\sin n}{n}\right)$$

$u_n \le v_n \le w_n \quad \forall \quad n \ge p \in \mathbb{Z}^+$

if $\{u_n\}$ & $\{w_n\}$ converge and

$$\underbrace{\lim_{n\to\infty} u_n}_{} = \underbrace{\lim_{n\to\infty} w_n}_{} = L$$

Then $\{v_n\}$ converges and $\lim_{n\to\infty} v_n = L$



Ex: $\lim_{n\to\infty} \frac{\sin n}{n}$

$-1 \le \sin n \le 1$ for all $n \in \mathbb{Z}^+$

$* \xrightarrow{\div n} \quad -\frac{1}{n} \le \frac{\sin n}{n} \le \frac{1}{n}$

$u_n = \frac{-1}{n}$

$\lim\left(\frac{-1}{}\right)$ ... $= \lim\left(\frac{1}{}\right)$

$$u_n = \frac{-1}{n}$$

$$\lim_{n\to\infty}\left(\frac{-1}{n}\right) = 0 = \lim_{n\to\infty}\left(\frac{1}{n}\right) \qquad w_n = \frac{1}{n}$$

$*$ $\xrightarrow[\lim_{n\to\infty}]{\text{Apply}}$ $\lim_{n\to\infty} \frac{-1}{n} \le \lim_{n\to\infty} \frac{\sin n}{n} \le \lim_{n\to\infty} \frac{1}{n}$

$$0 \le \lim_{n\to\infty} \frac{\sin n}{n} \le 0$$

$$\boxed{\lim_{n\to\infty} \frac{\sin n}{n} = 0} \leftarrow \text{convergent}$$

Ex:

$$\lim_{n\to\infty}\left(\frac{n!}{n^n}\right) \qquad \underline{u_n} \le v_n \le \underline{w_n}$$

$$\frac{n!}{n^n} = \frac{n}{n} \times \frac{n-1}{n} \times \frac{n-2}{n} \times \frac{n-3}{n} \cdots \frac{3}{n} \cdot \frac{2}{n} \frac{1}{n}$$

$$< \frac{n}{n} \frac{n}{n} \times \frac{n}{n} \times \frac{n}{n} \cdots \frac{n}{n} \times \frac{n}{n} \times \frac{1}{n} = \frac{1}{n}$$

$$0 < \frac{n!}{n^n} < \frac{1}{n} \quad \text{— for all } n \in \mathbb{Z}^+$$

$$\lim_{n\to\infty} 0 < \lim_{n\to\infty} \frac{n!}{n^n} < \lim_{n\to 0} \frac{1}{n}$$

$$0 < \lim_{n\to\infty} \frac{n!}{n^n} < 0$$

$$\lim_{n\to\infty} \frac{n!}{n^n} = 0$$

$$\lim_{n\to\infty} \frac{n!}{n^n} = 0$$

Ex

$$u_n = \frac{3n + \sin 2n}{4n-3} \quad --- \quad n \in \mathbb{Z}^+$$

$$\frac{3n-1}{4n-3} \leq \frac{3n+\sin(2n)}{4n-3} \leq \frac{3n+1}{4n-3}$$

$$\| \; 3/4$$

$$\lim_{n\to\infty} u_n = \frac{3}{4}$$

1) Prove that the sequence defined by $V_n = \dfrac{\sin(2n+1)}{n}$ is convergent.

2) Given the convergent sequence defined by

$$u_n = \begin{cases} u_1 = 1 \\ u_{n+1} = 1 + \dfrac{1}{u_n}, & n \in \mathbb{Z}^+ \end{cases}$$

find its limit

Divergent Sequences (indeterminate forms)

if $\lim_{n\to\infty} |u_n| = +\infty \Rightarrow \lim_{n\to\infty} \frac{1}{u_n} = 0$

$\lim_{n\to\infty} u_n = 0 \Rightarrow \lim \left| \frac{1}{u_n} \right| = +\infty$

$\lim_{n\to\infty} u_n = 0 \Rightarrow \lim_{n\to\infty} \left|\frac{1}{u_n}\right| = +\infty$

$$a_n = \frac{4n^2+1}{3n^2-1}$$

$$\lim_{n\to\infty} \frac{4n^2+1}{3n^2-1} \qquad \left(\frac{\infty}{\infty}\right) \text{ ind.}$$

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)$$

← Rational function. we check → $f(a) = g(a) = 0$

$\left(\frac{0}{0}\right)$ ind.

$$= \lim_{x\to a} \frac{f(x) - f(a)}{g(x) - g(a)}$$

$x \to a$

$x - a \neq 0$

$$= \lim_{x\to a} \frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}}$$

$$= \frac{\lim_{x\to a} \frac{f(x)-f(a)}{x-a}}{\lim_{x\to a} \frac{g(x)-g(a)}{x-a}} = \lim_{x\to a}\left(\frac{f'(x)}{g'(x)}\right)$$