Binomial Expansion
Practice questions

1

The function f is defined by

$$f: x \to \frac{x-2}{1+2x}, \quad x \in \mathbb{R},\ a < x < b.$$

Expand $f(x)$ in ascending powers of x up to and including the term in x^3. State the minimum value of a and the maximum value of b for the expansion to be valid. [3]

2

The series expansion of $(1+ax)^b$ up to and including the term in x^2 is given by $1-\frac{3}{2}x-\frac{3}{8}x^2$. Find the values of a and b. Explain why the substitution $x=-2$ may not be suitable in estimating the value of $5^{\frac{7}{4}}$ using the above series. [5]

3

(i) Expand $\frac{\sqrt{4+x}}{1-x}$ in ascending powers of x, up to and including the term in x^2. State the set of values of x for which the expansion is valid. [5]

(ii) By substituting $x=-\frac{1}{10}$, obtain an estimate for $\sqrt{390}$, leaving your answer as a fraction. [2]

4

Find the expansion of $\left(\frac{1+2x^2}{4-x}\right)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2. [4]

(i) Find the set of values of x for which the expansion is valid. [2]

(ii) By putting $x=\frac{1}{4}$, show that $\sqrt{30} \approx \frac{a}{b}$, where a and b are integers to be determined. [2]

5

By considering the expansion of $\frac{1}{\sqrt{1-x^2}}$, or otherwise, show that

$$\sin^{-1} x = x + \frac{x^3}{6} + ax^5 + \ldots$$

where a is a constant to be determined. [5]